# ON THE THEORY OF EXTRACTION FROM A FALLING DROPLET

## (K TEORII EKSTRAKTSII IZ PADAIUSHCHEI KAPLI)

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The problem of mass extraction from a spherical droplet of small radius falling under the action of gravity in a liquid medium was considered in [1]. It was assumed therein that the rate of extraction was limited by convective diffusion through the external medium, and the Peclet number for the external medium was small, so that the major change in the concentration occurred in the diffusion boundary layer in the neighborhood of the surface of the droplet. A similar problem was treated by Canadian authors [2 and 3], who made a trial-and-error calculation of the effect of the hydrodynamic flows inside and outside the droplet on the rate of mass transport. The difference between the latter papers and the paper [1] consisted on the one hand, in the use of an integral form of the expression for the stream function obtained by Hadamard [4] and Rybczynski [5], and on the other hand, in a considerably less exact solution of the equation of convective diffusion by trying various polynomial expressions for the concentration profile. The effect of convective transport on the rate of depletion of the solute within the falling droplet was investigated in a paper by Kronig and Brink [6] for the situation where the slow stage of the process was diffusion through the droplet. For estimating the maximum effect of the circulatory motion of the liquid within the droplet, the authors simplified the equation of mass transport, assuming that the concentration of solute within the droplet remained constant along each streamline. As shown by an estimate made by these same authors, such an assumption is certainly not satisfied near the surface of the droplet, so that the results obtained in [6] may not be applicable for large Peclet numbers, when the main resistance to mass transfer is concentrated in the diffusion boundary layer.

The problem of convective diffusion of mass from a single droplet moving in a liquid medium at small Reynolds numbers is considered herein, under the assumption that the Peclet numbers corresponding to the disperce (interior of the drop) and dense phases are very large and that the resistances of each of the phases to mass transfer are comparable. The problem is solved in the quasi-steady approximation, i.e. under the assumption that the relaxation time of the diffusion boundary layer is small in comparison with the time for a significant depletion of the solute in the droplet. The Poincaré-Lighthill-Kuo method is used to obtain the first two terms of the expansions of the mass-concentration functions for the interior and exterior regions in powers of certain parameters corresonding to these regions which are assumed to be of the same order of smallness. Expressions for the mass flux through the surface of the droplet and for the mean mass-transfer coefficients for each phase are found from the solutions obtained. One of the original assumptions of the problem was that there is interphase equilibrium expressed by Henry's law at the surface of the droplet, hence the total resistance to mass transfer (i.e. the reciprocal of the overall mass-transfer coefficient) was found to be equal to the sum of the resistances of each of the phases.

We shall consider a spherical droplet of sufficiently small radius $R$ containing solute and moving with a constant velocity $U$ in a liquid medium. Let $D_1$ and $D_2$ be, respectively, the mass-diffusion coefficients inside and outside the droplet. If the Peclet numbers $P_1 = UR/D_1$ and $P_2 = UR/D_2$ are sufficiently large, then the main resistance to mass transfer from the droplet will be concentrated in a thin layer called the diffusion boundary layer, which will be located on both sides of the surface of the droplet. It may be shown that for given values of the mass concentrations at the edges of the diffusion layer, the relaxation time of the latter, that is the time during which a steady distribution of concentration is established in the diffusion layer, is

$$\tau_r = \frac{R(1+\mu)}{2U} \qquad \left(\mu = \frac{\mu_1}{\mu_2}\right)$$

where $\mu$ is the ratio of the dynamic viscosities of the disperse and dense liquid phases. In [6] it was shown that the time $\tau_d$ during which the total quantity of solute in the drop decreases by the factor $e$ is bounded below by the quantity $0.022R^2/D_1$.

Consequently, we have the relation

$$\frac{\tau_r}{\tau_d} < 23\,\frac{1+\mu}{P_1}$$

which shows that at sufficiently large Peclet numbers $P_1$ there exists a sufficiently long interval of time $t$ satisfying the condition $\tau_r \ll t \ll \tau_d$. During this interval of time the mass transfer in the diffusion boundary layer may be described by the steady-state equation of convective diffusion for prescribed values of the concentrations at the edges of the boundary layer equal to the initial values of the concentrations inside and outside the drop. In this case the problem reduces to finding the solution of the system

$$\begin{aligned} \frac{v_r^{(1)}}{U}\frac{\partial c_1}{\partial r} + \frac{v_\theta^{(1)}}{Ur}\frac{\partial c_1}{\partial \theta} &= \frac{1}{\beta P_2}\Delta_{r,\theta}\, c_1 \\ \frac{v_r^{(2)}}{U}\frac{\partial c_2}{\partial r} + \frac{v_\theta^{(2)}}{Ur}\frac{\partial c_2}{\partial \theta} &= \frac{1}{P_2}\Delta_{r,\theta}\, c_2 \end{aligned} \qquad \left(\beta = \frac{D_2}{D_1}\right) \qquad (1)$$

where $\Delta_{r,\theta}$ is the Laplace operator in spherical coordinates, $r$ is the distance from the origin expressed in units of the droplet radius, and $\theta$ is the polar angle. If the origin is located at the center of the droplet, the polar axis directed vertically upwards, and the falling of the droplet at small Reynolds numbers under the action of gravity is considered, then, as shown in [4 and 5], the hydrodynamic velocity distributions within and outside the droplet will have respectively the following forms:

$$v_r^{(1)} = -\frac{U(1-r^2)\cos\theta}{2(1+\mu)}, \qquad v_\theta^{(1)} = \frac{U(1-2r^2)\sin\theta}{2(1+\mu)} \qquad (2)$$

$$v_r^{(2)} = U\left(1 - \frac{3a}{2r} + \frac{b}{2r^3}\right)\cos\theta, \qquad v_\theta^{(2)} = U\left(-1 + \frac{3a}{4r} + \frac{b}{4r^3}\right)\sin\theta \qquad (3)$$

$$\left(a = \frac{2+3\mu}{3(1+\mu)}, \qquad b = \frac{\mu}{1+\mu}\right)$$

As noted above, in the interval of time under consideration $(t \ll \tau_d)$, the mass concentration in the interior of the droplet (at the edge of the diffusion boundary layer) may be treated as constant and equal to the initial value $c_{10}$

$$c_1 (r \ll 1, \theta) = c_{10} \tag{4}$$

We will also assume that the mass concentration in the continuous medium at a large distance from the droplet is given

$$c_2 (r \gg 1, \theta) = c_{2\infty} \tag{5}$$

On the surface of the droplet the mass flux must be continuous

$$D_1 \frac{\partial c_1}{\partial r} (1, \theta) = D_2 \frac{\partial c_2}{\partial r} (1, \theta) \tag{6}$$

Moreover, it may be assumed that owing to the large rates of physical diffusion at the interface, equilibrium between phases is established instantaneously. For sufficiently small concentrations of the extracted substance, such a situation is described by Henry's law

$$c_1 (1, \theta) = \alpha c_2 (1, \theta) \tag{7}$$

where $\alpha$ is a coefficient depending on the temperature and pressure.

The Poincaré-Lighthill-Kuo method [7 to 11], which has found very wide and effective application in various problems of hydrodynamics, heat and mass transfer in recent years, will be used in the solution of the present problem. This method is a combination of Prandtl's boundary layer method of modifying perturbation theory due to Poincaré and Lighthill, in which the series expansion in powers of a small parameter is applied not only to the sought-for function, but also to the independent variable. The main advantage of this method is that it avoids an increase in the order of the singularity in the solutions corresponding to higher approximations, and it thereby eliminates divergence of integrals of quantities obtained from these solutions.

We shall choose for expansion parameters corresponding to the disperse and dense phases the quantities

$$f_1 = \sqrt{(1 + \mu) / \beta P_2}, \qquad f_2 = \sqrt{(1 + \mu) / P_2} \tag{8}$$

and we shall assume that these quantities are of the same order of smallness. It is easily seen that the condition $f_1 \ll 1$ ensures the fulfillment of the inequality $\tau_r \ll \tau_d$, which was already used in formulating the basic equations (1) of the diffusion boundary layer.

We write the expansions for the quantities $c_i$ and $r$ as power series in the parameters $f_i$ $(i = 1, 2)$, retaining first-order terms

$$c_i = c_i^{(0)} + f_i c_i^{(1)} + \ldots, \qquad r = 1 + f_i \rho + \ldots \tag{9}$$

Substituting these expansions into Equation (1) and the boundary conditions (4) to (7), we obtain for $c_i^{(0)}$ the system oe equations

$$\rho\nu \frac{\partial c_i^{(0)}}{\partial \rho} + \frac{1 - \nu^2}{2} \frac{\partial c_i^{(0)}}{\partial \nu} - \frac{\partial^2 c_i^{(0)}}{\partial \rho^2} = 0 \tag{10}$$

with boundary conditions

$$c_1^{(0)}(-\infty, \nu) = c_{10}, \qquad c_2^{(0)}(\infty, \nu) = c_{2\infty}$$
$$c_1^{(0)}(0, \nu) = \alpha c_2^{(0)}(0, \nu), \quad \frac{\partial c_1^{(0)}}{\partial \rho}(0, \nu) = \sqrt{\beta}\,\frac{\partial c_2^{(0)}}{\partial \rho}(0, \nu) \quad (\nu = \cos\theta) \tag{11}$$

It is not difficult to show that this system has the solution (12)

$$c_i^{(0)} = A_i \operatorname{erf}\chi + B_i \quad (i = 1,2),\ \chi(\rho, \nu) = \rho(1 - \nu^2)\left[8\int_{-1}^{\nu}(1 - x^2)\,dx\right]^{-1/2}$$

$$A_1 = -\frac{\sqrt{\beta}(c_{10} - \alpha c_{2\infty})}{\alpha + \sqrt{\beta}}, \qquad A_2 = -\frac{c_{10} - \alpha c_{2\infty}}{\alpha + \sqrt{\beta}}$$
$$B_1 = \frac{\alpha\left(c_{10} + \sqrt{\beta}c_{2\infty}\right)}{\alpha + \sqrt{\beta}}, \qquad B_2 = \frac{c_{10} + \sqrt{\beta}\,c_{2\infty}}{\alpha + \sqrt{\beta}} \tag{13}$$

If the system of equations for $c_i^{(1)}$ were now written down, then the correction to the expression for the local mass flux at the interface obtained by solving this system would have singularities at $\nu = \pm 1$, leading to divergence of the mass-flux integral. To avoid this difficulty we introduce the coordinate transformation

$$\rho = \xi, \qquad \nu = \eta + f_i g_i(\xi, \eta) \tag{14}$$

selecting the functions $g_i(\xi, \eta)$ in such a way that the correction to the local flux at the interface, calculated to first order in the parameters $f_i$, will not contain undesirable singularities. It is obvious that the change of variables from $\rho$, $\nu$ to $\xi$, $\eta$ as defined in Equations (14) can alter only the equations for $c_i^{(1)}$. The solution for $c_i^{(0)}$, however, will as before be given by Formulas (12) and (13) within the accuracy of the substitution $(\rho, \nu) \to (\xi, \eta)$.

Introducing the change of coordinates (14) into Equations (1) and substituting the expansion (9), we obtain the system (15)

$$\xi\eta\frac{\partial c_i^{(1)}}{\partial \xi} + \frac{1 - \eta^2}{2}\frac{\partial c_i^{(1)}}{\partial \eta} - \frac{\partial^2 c_i^{(1)}}{\partial \xi^2} = \left(2 - \xi g_i - \frac{\xi^2\eta}{2}h_i\right)\frac{\partial c_i^{(0)}}{\partial \xi} +$$
$$+\left[\eta\left(g_i + \xi\frac{\partial g_i}{\partial \xi}\right) + \frac{1 - \eta^2}{2}\left(\frac{\partial g_i}{\partial \eta} - 3\xi k_i\right) - \frac{\partial^2 g_i}{\partial \xi^2}\right]\frac{\partial c_i^{(0)}}{\partial \eta} - 2\frac{\partial g_i}{\partial \xi}\frac{\partial^2 c_i^{(0)}}{\partial \xi\,\partial \eta} \quad (i=1,2)$$

with boundary conditions

$$c_1^{(1)}(-\infty, \eta) = c_2^{(1)}(\infty, \eta) = 0, \quad c_1^{(1)}(0, \eta) = \alpha\sqrt{\beta}\,c_2^{(1)}(0, \eta)$$
$$\frac{\partial c_1^{(1)}}{\partial \xi}(0, \eta) = \beta\frac{\partial c_2^{(1)}}{\partial \xi}(0, \eta) \quad (h_1 = k_1 = 1, \quad h_2 = 3\mu - 2, \quad k_2 = \mu) \tag{16}$$

We also introduce new variables $\psi$ and $\tau$ by Formulas

$$\psi = \xi(1 - \eta^2), \qquad \tau = 2\int_{-1}^{\eta}(1 - x^2)\,dx = \frac{2}{3}(2 - \eta)(1 + \eta)^2 \tag{17}$$

Transforming Equation (15) to the variables (17) and substituting Expressions (12) and (13) for the functions $c_i^{(0)}$, we obtain an equation similar to the equation of heat conduction with sources

$$\frac{\partial c_i^{(1)}}{\partial\tau} - \frac{\partial^2 c_i^{(1)}}{\partial\psi^2} = \frac{A_i}{\sqrt{\pi\tau}} e^{-\psi^2/4\tau}\left\{\frac{2}{1-\eta^2} + \frac{\psi^2\eta\,(6k_i - h_i)}{2\,(1-\eta^2)^3} - \frac{\psi g_i}{1-\eta^2}\left[\frac{\eta}{\tau} + \frac{1+\eta^2}{(1-\eta^2)^2}\right] + \right.$$
$$\left. + \frac{3}{2}\frac{\psi^2 k_i}{\tau\,(1-\eta^2)} + \frac{2\psi}{1-\eta^2}\left[\eta + \frac{(1-\eta^2)^2}{2\tau}\right]\left[\frac{\partial^2 g_i}{\partial\psi^2} - \frac{\partial g_i}{\partial\tau} + \frac{2}{\psi}\left(1 - \frac{\psi^2}{2\tau}\right)\frac{\partial g_i}{\partial\psi}\right]\right\} \tag{18}$$

We choose the functions $g_i(\psi, \eta)$ so that the terms on the right-hand side of Equation (18) having strong singularities at the points $\eta = \pm 1$ will vanish. For this purpose it is necessary to set

$$g_i(\psi, \eta) = \frac{\psi\eta\,(6k_i - h_i)}{2\,(1+\eta^2)} \tag{19}$$

for which Equation (18) takes the form

$$\frac{\partial c_i^{(1)}}{\partial\tau} - \frac{\partial^2 c_i^{(1)}}{\partial\psi^2} = \frac{2A_i}{\sqrt{\pi\tau}\,(1-\eta^2)} e^{-\psi^2/4\tau}\left\{1 + \frac{3}{4}\frac{\psi^2}{\tau}k_i - \frac{\psi^2(6k_i - h_i)}{4\,(1+\eta^2)}\left[\frac{3\eta^2}{\tau} + \right.\right.$$
$$\left.\left. + \frac{\eta}{1+\eta^2} + \frac{(1-\eta^2)^2}{2\tau}\left(\frac{1}{1+\eta^2} + \frac{2\eta}{\tau}\right)\right] + \frac{\eta(6k_i - h_i)}{1+\eta^2}\left[\eta + \frac{(1-\eta^2)^2}{2\tau}\right]\right\} \quad (i=1,\ 2) \tag{20}$$

where the function $\eta(\tau)$ is determined by the relation (17).

It is necessary to add still another condition in order to obtain the solution of Equation (20) with the boundary conditions (16). For this we may require [1] that the mass concentration at the stagnation point ($\tau = 0$) be equal to the value of the concentration far from the droplet, i.e. $c_{2\infty}$ Thus Equation (20) must be solved with boundary conditions

$$c_1^{(1)}(-\infty, \tau) = c_2^{(1)}(\infty, \tau) = 0, \qquad c_1^{(1)}(\psi, 0) = c_2^{(1)}(\psi, 0) = 0$$
$$c_1^{(1)}(0, \tau) = \alpha\sqrt{\beta}\,c_2^{(1)}(0, \tau), \qquad \frac{\partial c_1^{(1)}}{\partial\psi}(0, \tau) = \beta\,\frac{\partial c_2^{(1)}}{\partial\psi}(0, \tau) \tag{21}$$

We will seek the solution in the form of a sum

$$c_i^{(1)} = Y_i^{(1)} + Y_i^{(2)} \qquad (i = 1,\ 2) \tag{22}$$

We require that the functions $Y_i^{(1)}(\psi, \tau)$ satisfy Equation (20) with zero initial conditions. Then, as is known

$$Y_1^{(1)}(\psi, \tau) = -\int_0^\tau\int_0^\infty \frac{F_1(x, y)}{\sqrt{4\pi(\tau-y)}}\left\{\exp\left[-\frac{(\psi-x)^2}{4(\tau-y)}\right] - \exp\left[-\frac{(\psi+x)^2}{4(\tau-y)}\right]\right\}dx\,dy$$
$$Y_2^{(1)}(\psi, \tau) = \int_0^\tau\int_0^\infty \frac{F_2(x, y)}{\sqrt{4\pi(\tau-y)}}\left\{\exp\left[-\frac{(\psi-x)^2}{4(\tau-y)}\right] - \exp\left[-\frac{(\psi+x)^2}{4(\tau-y)}\right]\right\}dx\,dy \tag{23}$$

Here each of the functions $F_i(\varphi, \tau)$ is the corresponding right-hand side of Equation (20).

The functions $Y_i^{(2)}(\psi, \tau)$ must satisfy the homogeneous heat-conduction equation with boundary conditions

$$Y_1^{(2)}(-\infty, \tau) = 0, \quad Y_2^{(2)}(\infty, \tau) = 0, \quad Y_1^{(2)}(\psi, 0) = 0, \quad Y_2^{(2)}(\psi, 0) = 0 \tag{24}$$

$$Y_1^{(2)}(0, \tau) = \alpha\sqrt{\beta}\, Y_2^{(2)}(0, \tau) \tag{25}$$

$$\frac{\partial Y_1^{(2)}}{\partial\psi}(0, \tau) - \beta\frac{\partial Y_2^{(2)}}{\partial\psi}(0, \tau) = \int_0^{\tau}\int_0^{\infty}\frac{F_1(x, y) + \beta F_2(x, y)}{\sqrt{4\pi(\tau - y)^3}}\, x \exp\left[-\frac{x^2}{4(\tau - y)}\right] dx\, dy \tag{26}$$

The expressions for the functions $Y_i^{(2)}(0, \tau)$, taking account of the boundary conditions (24), may by means of Duhamel's integral be represented in the form

$$Y_1^{(2)}(0, \tau) = \int_0^{\tau}\left[\frac{\partial Y_1^{(2)}}{\partial\psi}\right]_{\substack{\psi=0\\ \tau=y}}\frac{dy}{\sqrt{\pi(\tau - y)}}$$

$$Y_2^{(2)}(0, \tau) = -\int_0^{\tau}\left[\frac{\partial Y_2^{(2)}}{\partial\psi}\right]_{\substack{\psi=0\\ \tau=y}}\frac{dy}{\sqrt{\pi(\tau - y)}} \tag{27}$$

Using the boundary condition (25), we obtain

$$\int_0^{\tau}(\tau - y)^{-1/2}\left(\left[\frac{\partial Y_1^{(2)}}{\partial\psi}\right]_{\substack{\psi=0\\ \tau=y}} + \alpha\sqrt{\beta}\left[\frac{\partial Y_2^{(2)}}{\partial\psi}\right]_{\substack{\psi=0\\ \tau=y}}\right)dy = 0 \tag{28}$$

Hence it follows that

$$\left[\frac{\partial Y_1^{(2)}}{\partial\psi}\right]_{\psi=0} + \alpha\sqrt{\beta}\left[\frac{\partial Y_2^{(2)}}{\partial\psi}\right]_{\psi=0} = 0 \tag{29}$$

Solving Equations (29) and (26) simultaneously, we find expressions for the derivatives (30)

$$\left[\frac{\partial Y_1^{(2)}}{\partial\psi}\right]_{\psi=0} = \frac{\alpha}{\alpha + \sqrt{\beta}}\int_0^{\tau}\int_0^{\infty}\frac{F_1(x, y) + \beta F_2(x, y)}{\sqrt{4\pi(\tau - y)^3}}\, x \exp\left[-\frac{x^2}{4(\tau - y)}\right] dx\, dy$$

$$\left[\frac{\partial Y_2^{(2)}}{\partial\psi}\right]_{\psi=0} = -\frac{1}{\beta + \alpha\sqrt{\beta}}\int_0^{\tau}\int_0^{\infty}\frac{F_1(x, y) + \beta F_2(x, y)}{\sqrt{4\pi(\tau - y)^3}}\, x \exp\left[-\frac{x^2}{4(\tau - y)}\right] dx\, dy$$

The mass flux at a given point of the surface of the droplet is determined by Expression

$$j(\nu) = -\frac{D_i}{R}\frac{\partial c_i}{\partial r}(1, \nu) = -\frac{D_i}{Rf_i}(1 - \nu^2)\left[\frac{\partial c_i^{(0)}}{\partial\psi}\right]_{\psi=0} - \\ - \frac{D_i}{R}(1 - \nu^2)\left[\frac{\partial Y_i^{(1)}}{\partial\psi} + \frac{\partial Y_i^{(2)}}{\partial\psi}\right]_{\psi=0} \tag{31}$$

Here the derivatives $[\partial c_i^{(0)} / \partial\psi]_{\psi=0}$, $[\partial Y_i^{(1)} / \partial\psi]_{\psi=0}$, $[\partial Y_i^{(2)} / \partial\psi]_{\psi=0}$ will be known functions of the variable $\tau = \tau(\eta)$. According to (19) the function $\sigma_i(\psi, \eta)$, which determines the angle transformation, vanishes on the surface of the drop, i.e. for $\psi = 0$, hence for the inverse transformation from the variable $\tau(\eta)$ to the variable $\nu$ in Formula (31), it suffices to replace $\eta$ by $\nu$ in all of the functions. The total mass flow through the surface of the droplet equals

$$I = 2\pi R^2 \int_{-1}^{1} j(\nu)\, d\nu = \frac{2RD_2(c_{10} - \alpha c_{2\infty})}{\alpha + \sqrt{\beta}} \left\{ \left( \frac{8\pi P_2}{3(1+\mu)} \right)^{1/2} - \frac{1}{\alpha + \sqrt{\beta}} I_1 \right\} \tag{32}$$

where

$$I_1 = \frac{\alpha + \sqrt{\beta}}{2\sqrt{\beta}} \int_{-1}^{1} (1 - \nu^2)\, d\nu \int_0^{\tau(\nu)} \int_0^{\infty} \frac{\alpha \sqrt{\beta} F_2(x, y) - F_1(x, y)}{(\tau - y)^{3/2}} x \exp\left[ - \frac{x^2}{4(\tau - y)} \right] dx\, dy \tag{33}$$

The relation $\tau(\nu)$ is determined by (17). As a result of some calculations Formula (32) takes the form

$$I = \frac{2RD_2(c_{10} - \alpha c_{2\infty})}{\alpha + \sqrt{\beta}} \left\{ \left( \frac{8\pi P_2}{3(1+\mu)} \right)^{1/2} - \frac{5.5 - \alpha(3.8 + 1.7\mu)}{\alpha + \sqrt{\beta}} \right\} \tag{34}$$

The initial assumptions that the quantities $f_1$ and $f_2$ were small and of the same order do not allow the use of Formula (34) over a wide range of the parameters appearing in it. We note, however, that in the limiting case $D_1 \to \infty$ ($\beta \to 0$), when the rate of mass transfer is limited by the external medium, considering the interval of time to be small in comparison with the time of depletion of the drop, the mass concentration at the surface of the droplet can be considered as fixed at the value $c_{10}$. In this case the problem reduces to the solution of the second equation of (1) alone, which does not contain the parameter $\beta$, and the expression for the mass flow takes the form

$$I = 2RD_2(c_{10} - c_{2\infty}) \left\{ \left( \frac{8\pi P_2}{3(1+\mu)} \right)^{1/2} - 1.7(1+\mu) \right\} \tag{35}$$

Expression (35) is the formula of Levich [1] taking into account subsequent nonvanishing terms in the expansion for the concentration $c_2$ in powers of the parameter $f_2$. It should, however, be noted that the main advantage of Formula (34) over Levich's formula, besides the fact that it is applicable to a wider range of Peclet numbers, is that it explicitly accounts (through $\alpha$) for the dependence of the flow $I$ on the physical properties of the contiguous phases, and that there are no restrictions on the absolute value of the coefficient $\alpha$ appearing in this formula.

Formula (34) determines the overall mass transfer coefficient

$$K = \frac{I}{4\pi R^2 (c_{10} - \alpha c_{2\infty})}$$

The partial mass transfer coefficients $K_1$ and $K_2$ corresponding to each phase are defined by the relations

$$K_1 = \frac{I}{4\pi R^2} \left[ c_{10} - \frac{1}{2} \int_{-1}^{1} c_1(1, \nu)\, d\nu \right]^{-1}$$

$$K_2 = \frac{I}{4\pi R^2} \left[ \frac{1}{2\alpha} \int_{-1}^{1} c_1(1, \nu)\, d\nu - c_{2\infty} \right]^{-1} \tag{36}$$

where the function $c_1(1, \nu)$, which is the value of the concentration $c_1$ on the surface of the droplet, has, according to Formulas (12). (13), (22), (23), (27) and (30) for $Y_1$ the following form:

$$c_1(1, \nu) = \frac{\alpha(c_{10} + \sqrt{\beta}\, c_{2\infty})}{\alpha + \sqrt{\beta}} + \frac{\alpha f_1}{2\pi(\alpha + \sqrt{\beta})} \int_0^{\tau(\nu)} \frac{dy}{\sqrt{\tau - y}} \int_0^y \int_0^\infty \frac{F_1(x,\zeta) + \beta F_2(x,\zeta)}{(y-\zeta)^{3/2}} \times$$

$$\times\, x \exp\left[-\frac{x^2}{4(y-\zeta)}\right] dx\, d\zeta \qquad (37)$$

Substituting this expression into Formulas (36), we obtain

$$K_1 = \frac{I(\alpha + \sqrt{\beta})}{4\pi R^2 \sqrt{\beta}(c_{10} - \alpha c_{2\infty})} \left[1 + \frac{\alpha}{\alpha + \sqrt{\beta}} \left(\frac{1+\mu}{P_1}\right)^{1/2} I_2\right]^{-1} \qquad (38)$$

$$K_2 = \frac{I(\alpha + \sqrt{\beta})}{4\pi R^2 (c_{10} - \alpha c_{2\infty})} \left[1 - \frac{1}{\alpha + \sqrt{\beta}} \left(\frac{1+\mu}{P_2}\right)^{1/2} I_2\right]^{-1}$$

Here

$$I_2 = -\frac{\alpha + \sqrt{\beta}}{4\pi\sqrt{\beta}(c_{10} - \alpha c_{2\infty})} \int_{-1}^{1} d\nu \int_0^{\tau(\nu)} \frac{dy}{\sqrt{\tau - y}} \int_0^y \frac{d\zeta}{(y-\zeta)^{3/2}} \int_0^\infty [F_1(x, \zeta) +$$

$$+\, \beta F_2(x, \zeta)]\, x \exp\left[-\frac{x^2}{4(g-\zeta)}\right] dx \qquad (39)$$

Calculation leads to the following result:

$$I_2 = 4.9 + 2.4\sqrt{\beta}(1 + \mu) \qquad (40)$$

By comparison of Formulas (3.8) with the expression for the overall coefficient $K$, it is easily seen that under the conditions of the present problem the law of addition of the resistances of the phases is valid

$$1/K = 1/K_1 + \alpha/K_2$$

This conclusion is a direct consequence of the linear character of the condition of interphase equilibrium (7) used above. For a nonlinear condition, however, the relation between the coefficients $K_1$, $K_2$ and $K$ can be obtained only as a result of the correct solution of the corresponding problem of the distribution of concentrations. At the present time analytical methods for solving such a problem are lacking.

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